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Electrical conductivity of dense non-ideal plasmas in external HF electric field

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Abstract
In this work, the previously developed method for calculation of HF electroconductivity of non-ideal plasma is applied to the area of high electron densities: $10^{21} \leqslant N_e \leqslant 10^{23}$ cm^{-3} and in the temperature range 20 000 K $\leqslant T \leqslant$ 1000 000 K. This is enabled by the improvement of the numerical procedure. The real and imaginary parts of the HF conductivity are parametrized in the Drude–Lorentz form. One of the parameters is the static conductivity σ_0. The computations are carried out in the frequency range $[0, 0.05\omega_p]$, ω_p being the plasma frequency.

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This work is a continuation of the works [1, 2]. In [1] we presented data for slightly non-ideal plasma HF conductivity, while in [2] we have covered the area of moderately non-ideal plasmas. Here we present the data for strongly non-ideal plasmas with $\Gamma = \beta e^2/a \in [0.03\text{–}6.30]$. As usual, β here is the inverse temperature in energy units and a is the electronic Wigner–Seitz radius.

In this work a completely ionized hydrogen plasma is considered in a homogeneous and monochromatic HF external electric field

$$\vec{E}(t) = \vec{E}_0 \exp\{-\mathrm{i}\omega t\}.$$

The dynamic electric conductivity $\sigma(\omega)$ is given by a complex function of the field frequency:

$$\sigma(\omega) = \sigma_{\mathrm{Re}}(\omega) + \mathrm{i}\sigma_{\mathrm{Im}}(\omega), \tag{1}$$

and, according to [1, 2], $\sigma(\omega)$ is taken in the integrated Drude-like form,

$$\sigma(\omega) = \frac{4e^2}{3m} \int_0^\infty \frac{\tau(E)}{1 - \mathrm{i}\omega\tau(E)} \left[-\frac{\mathrm{d}w(E)}{\mathrm{d}E}\right] \rho(E) E \, \mathrm{d}E, \tag{2}$$

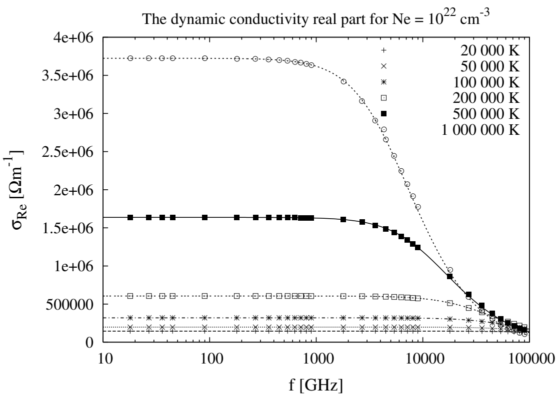

Figure 1. The dynamic conductivity real part σ_{Re} for $N_{\mathrm{e}} = 10^{22}$ cm^{-3}. Calculated values are presented by the corresponding symbols and the lines represent the values fitted according to equation (4).

where $\rho(E)$ is the density of electronic states in the energy space, $w(E)$ is a Fermi–Dirac distribution function and $\tau(E)$ is the static electronic relaxation time. The basic feature of our theory [3–6] is the evaluation of the relaxation time within the following approach: each electron (carrier) moves in a self-consistent field generated by all other free charges in the system. The finite values of the transport coefficients result from electron scattering on the self-consistent field fluctuations. It is based on the paper [7], which related the Lorenz-model expression for the fully-ionized plasma electrical conductivity to the strict quantum-statistical calculation involving Green's function formalism with the self-consistent field potential. It was shown that the static conductivity thus obtained is in semi-quantitative agreement with available experimental data and also possesses the correct limiting forms of Ziman and Spitzer, corresponding to high and low densities, respectively [6]. A detailed comparison with alternative methods of theoretical investigation of the dynamic conductivity, see, e.g., [8, 9], is beyond the scope of this paper.

Both σ_{Re} and σ_{Im} are computed in the area of electronic densities $10^{21} \leqslant N_{\mathrm{e}} \leqslant 10^{23}$ cm^{-3}, in the temperature range $20\,000$ K $\leqslant T \leqslant 1000\,000$ K. The results are displayed in figures 1–4. In these figures, the values of $\sigma_{\mathrm{Re}}(\omega)$ and $\sigma_{\mathrm{Im}}(\omega)$ are presented (by dotted lines) in the range $\omega \leqslant 0.05\,\omega_{\mathrm{p}}$, where ω_{p} is the plasma frequency.

In addition, following the previous papers [1, 2, 4], the possibility is examined here of reducing the dynamic conductivity to the Drude–Lorentz form by presenting our results on $\sigma_{\mathrm{Re}}(\omega)$ and $\sigma_{\mathrm{Im}}(\omega)$ in the factorized form,

$$\sigma_{\mathrm{Re}}(\omega) = \sigma_0 \frac{1}{1 + (\omega\tau_0^*)^2 k_1^2}, \qquad \sigma_{\mathrm{Im}}(\omega) = \sigma_0 \frac{(\omega\tau_0^*)k_2}{1 + (\omega\tau_0^*)^2 k_1^2}, \tag{3}$$

where σ_0 is the static electric conductivity and τ_0^* is the effective relaxation time defined by the relation: $\sigma_0 = \tau_0^* N_{\mathrm{e}} e^2/m$. The factors k_1 and k_2 describe here the influence of the deviation of the $\mathrm{d}w/\mathrm{d}E$ from the δ-function. If both k_1 and k_2 tend to 1, then equations (3) become expressions for the HF electric conductivity in the Drude model. With the help of the dimensionless parameter $f_{0\mathrm{p}} \equiv \omega_{\mathrm{p}}\tau_0^* = 4\pi\sigma_0/\omega_{\mathrm{p}}$ the expressions for $\sigma_{\mathrm{Re}}(\omega)$ and $\sigma_{\mathrm{Im}}(\omega)$ turn into

$$\sigma_{\mathrm{Re}}(\omega) = \sigma_0 \frac{1}{1 + \left(\frac{\omega}{\omega_{\mathrm{p}}}\right)^2 f_{0p}^2 k_1^2}, \qquad \sigma_{\mathrm{Im}}(\omega) = \sigma_0 \frac{\left(\frac{\omega}{\omega_{\mathrm{p}}}\right) f_{0p} k_2}{1 + \left(\frac{\omega}{\omega_{\mathrm{p}}}\right)^2 f_{0p}^2 k_1^2}, \tag{4}$$

where the static conductivity σ_0 is calculated within the self-consistent field model of [3–5].

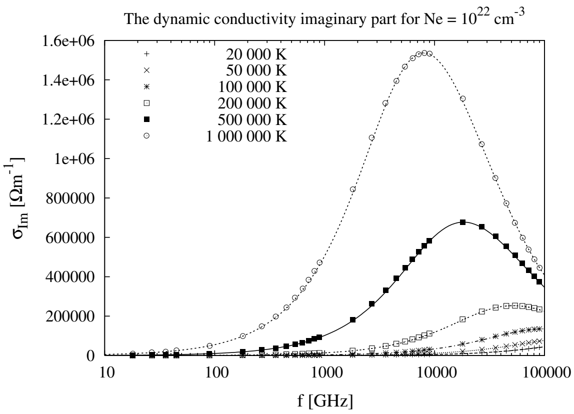

Figure 2. The dynamic conductivity imaginary part $\sigma_{\rm Re}$ for $N_{\rm e} = 10^{22}$ cm^{-3}. Calculated values are presented by the corresponding symbols and the lines represent the values fitted according to equation (4).

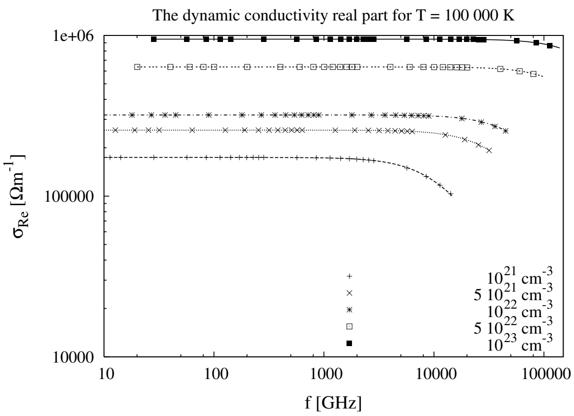

Figure 3. The dynamic conductivity real part $\sigma_{\rm Re}$ for $T = 100\,000$ K. Calculated values are presented by the corresponding symbols and the lines represent the values fitted according to equation (4).

The corresponding values of σ_0 and $f_{0\rm p}$ are presented in tables 1 and 2. The values of the factors k_1 and k_2 were determined from equations (2) and (4). Then, these factors can be fitted in the following approximate form,

$$k_1\left(\frac{\omega}{\omega_{\rm p}}\right) = k_{10} - \frac{a_1^2 b_1\left(\frac{\omega}{\omega_{\rm p}}\right)}{1 + a_1 b_1\left(\frac{\omega}{\omega_{\rm p}}\right)}, \qquad k_2\left(\frac{\omega}{\omega_{\rm p}}\right) = k_{20} - \frac{a_2^2 b_2\left(\frac{\omega}{\omega_{\rm p}}\right)}{1 + a_2 b_2\left(\frac{\omega}{\omega_{\rm p}}\right)}, \quad (5)$$

where the adjustment parameters k_{10}, a_1, b_1 and k_{20}, a_2, b_2 are determined numerically from the condition of the best agreement of the exact and the fitted values of the factors k_1 and k_2. The quality of the obtained fit is illustrated in figures 1–4, where the solid curves represent the behaviour of $\sigma_{\rm Re}$ and $\sigma_{\rm Im}$ as determined by (4) and (5).

In all these figures the quality of approximation (5) is confirmed: a very good agreement between the exact and fitting calculations is achieved in all domains of plasma parameters considered. Figures 1 and 2 illustrate the changes in the dynamic conductivity of dense plasma

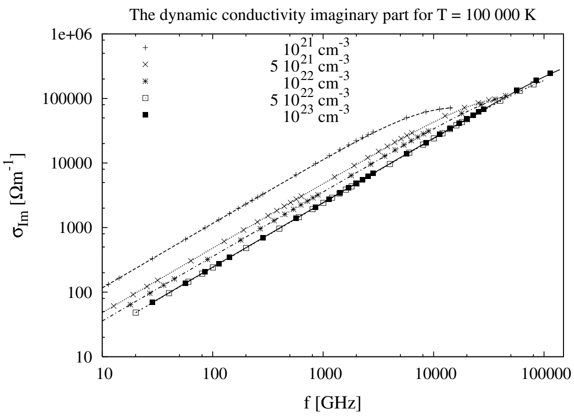

Figure 4. The dynamic conductivity imaginary part $\sigma_{\rm Re}$ for $T = 100\,000$ K. Calculated values are presented by corresponding symbols and the lines represent the values fitted according to equation (4).

Table 1. Static conductivity σ_0 versus the plasma electronic density $N_{\rm e}$ and temperature T $(10^3(\Omega\,{\rm m})^{-1})$.

	$N_{\rm e}$ (cm^{-3})				
T (K)	10^{21}	5×10^{21}	10^{22}	5×10^{22}	10^{23}
20 000	40.14	93.26	145.1	430.5	722.6
30 000	54.15	108.1	160.7	464.2	761.9
40 000	69.31	125.9	178.1	489.8	795.3
50 000	85.3	145.6	198.3	511.4	823.5
100 000	174	257.3	319.8	636.1	947.9
200 000	382.5	516.3	606.1	985.1	1304
500 000	1164	1460	1637	2276	2724
1000 000	2804	3390	3724	4825	5526

Table 2. Dimensionless parameter $f_{0\rm p}$ versus $N_{\rm e}$ and T.

	$N_{\rm e}$ (cm^{-3})				
T (K)	10^{21}	5×10^{21}	10^{22}	5×10^{22}	10^{23}
20 000	2.544 91	2.644 57	2.908 48	3.860 1	4.581 57
30 000	3.433 35	3.066 44	3.221 52	4.162 64	4.831 13
40 000	4.395 05	3.570 7	3.571 67	4.391 69	5.043 03
50 000	5.408 46	4.127 96	3.975 44	4.585 33	5.221 62
100 000	11.03	7.297 13	6.412 96	5.703 91	6.010 38
200 000	24.25	14.64	12.15	8.833 64	8.268 84
500 000	73.82	41.39	32.83	20.41	17.27
1000 000	177.8	96.12	74.67	43.26	35.04

with varying temperature, while figures 3 and 4 illustrate the behaviour of plasma at a constant temperature and for various values of the electronic density $N_{\rm e}$. The values of the fitting coefficients k_{10}, k_{20}, a_1, b_1, a_2 and b_2 vary slowly with the change of $N_{\rm e}$ and T, and thus can be easily interpolated for adequate values of electronic density and temperature within the investigated range. This parametrization could be useful in applications.

Acknowledgments

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